Automata Theory and Formal Grammars: Lecture 1

Sets, Languages, Logic

Sets, Languages, Logic

Today

- Course Overview
- Administrivia
- Sets Theory (Review?)
- Logic, Proofs (Review?)
- Words, and operations on them: $w_1 \circ w_2, w^i, w^*, w^+$
- Languages, and operations on them: $L_1 \circ L_2, L^i, L^*, L^+$

What This Course Is About

Mathematical theory of computation!

- We'll study different "machine models" (finite automata, pushdown automata)...
- ...with a view toward characterizing what they can compute.

Why Study This Topic?

- To understand the limits of computation.

  Some things require more resources to compute, and others cannot be computed at all. To study these issues we need mathematical notions of “resource” and “compute”.

- To learn some programming tools.

  Automata show up in many different settings: compilers, text editors, communications protocols, hardware design, . . .

  First compilers took several person-years; now written by a single student in one semester, thanks to theory of parsing.

- To learn about program analysis.

  Microsoft is shipping two model-checking tools. PREfix discovered $\geq$2000 bugs in XP (fixed in SP2).

- To learn to think analytically about computing.

Why Study This Topic?

- This course focuses on machines and logics.

  Analysis technique: model checking (SE431).

- CSC535 focuses on languages and types.

  Analysis technique: type checking (CSC535).

- Both approaches are very useful.

  For example, in Computer Security (SE547).

Administrivia

- Course Homepage:

  http://www.depaul.edu/~jriely/csc444fall2003/

- Syllabus:

  http://www.depaul.edu/~jriely/csc444fall2003/syllabus.html

Set Theory: Sets, Functions, Relations

Sets

Sets are collections of objects.

- $\{\ \}$, $\{42\}$, $\{\text{alice}, \text{bob}\}$
- $\mathbb{N} = \{0, 1, 2, ...\}$
- $\mathbb{Z} = \{..., -2, -1, 0, 1, 2, ...\}$
- $\mathbb{R} =$ the set of real numbers includings $\mathbb{Z}$, $\sqrt{2}$, π, etc
- $\{\, x \in \mathbb{N} \mid x \geq 5 \,\}$

Sets are unordered and insensitive to repetition.

- $\{42, 27\} = \{27, 42\}$
- $\{42, 42\} = \{42\}$

What Do the Following Mean?

$\emptyset, \{\}$	empty set
$a \in A$	membership
$A \subseteq B$	subset
$A \cup B$	union
$A \cap B$	intersection
$\circ A$	complement
$A - B$	set difference $= A \cap \circ B$
$\bigcup_{i \in I} A_i$	indexed union
$\bigcap_{i \in I} A_i$	indexed intersection
2^A	power set (set of all subsets)
$A \times B$	Cartesian product $= \{ \langle a, b \rangle \mid a \in A, b \in B \}$
$\lvert A \rvert$	size (cardinality, or number of elements)