Lecture 5

Partial Redundancy Elimination

I Forms of redundancy
- -- global common subexpression elimination
- -- loop invariant code motion
- -- partial redundancy

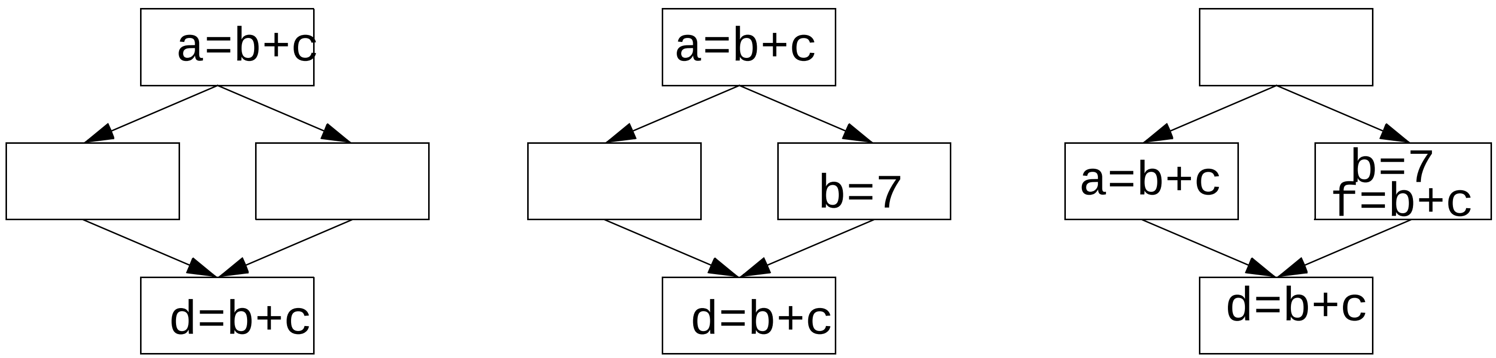
II Lazy Code Motion Algorithm

Reading: Chapter 9.5

Overview

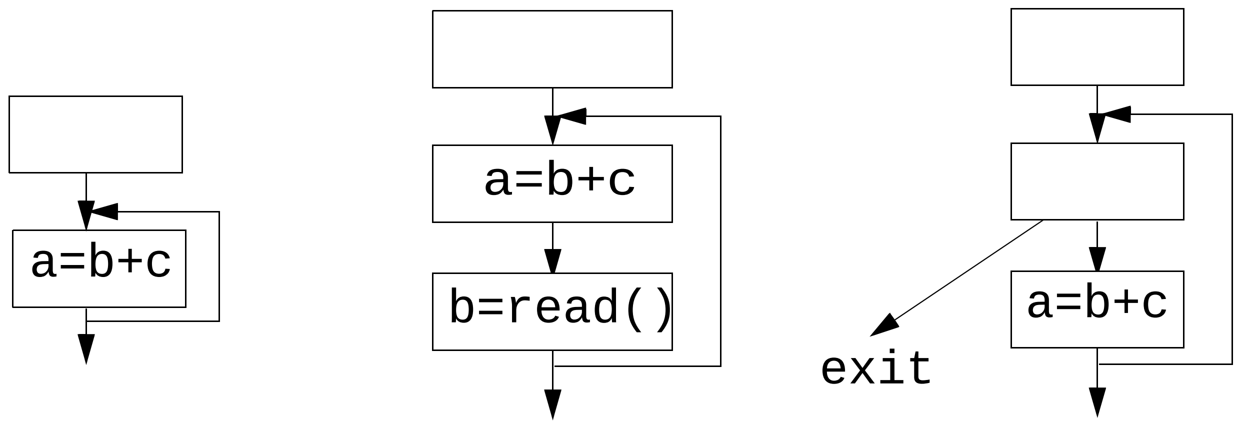
- **Eliminates many forms of redundancy in one fell swoop**
- **Originally formulated as 1 bi-directional analysis**
- **Lazy code motion algorithm**
    - formulated as 4 separate uni-directional passes (backward, forward, forward, backward)

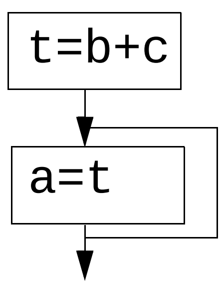
I. Common Subexpression Elimination



- **A common expression may have different values on different paths!**
- **On every path reaching p,**
  - expression b+c has been computed
  - b, c not overwritten after the expression

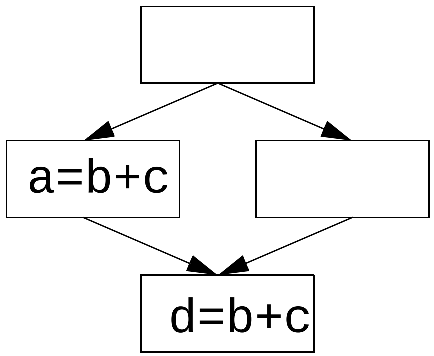
Loop Invariant Code Motion





- Given an expression (b+c) inside a loop,
  does the value of b+c change inside the loop?
  is the code executed at least once?

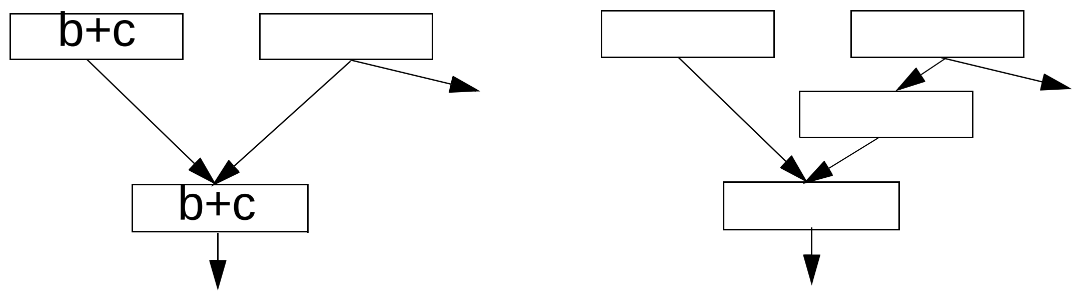
Partial Redundancy



- Can we place calculations of b+c
  such that no path re-executes the same expression

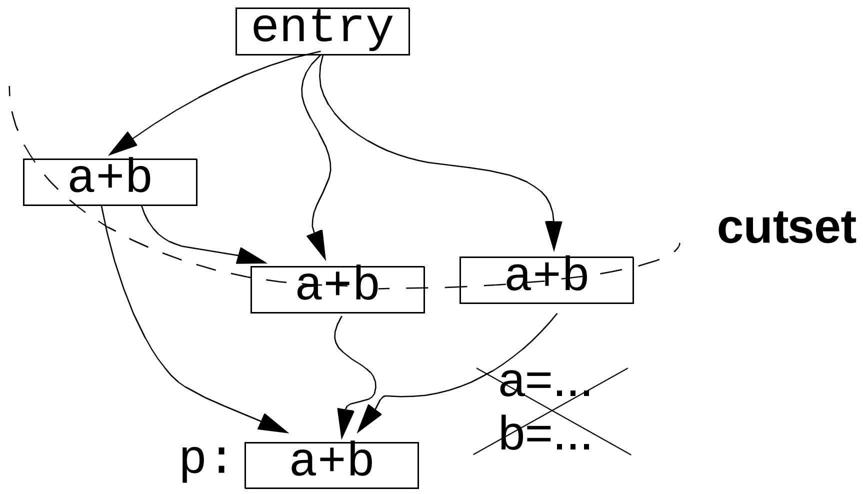
- Partial redundancy elimination (PRE)
  - subsumes:
    - global common subexpression (full redundancy)
    - loop invariant code motion (partial redundancy for loops)

II. Increasing the Chance of Optimization



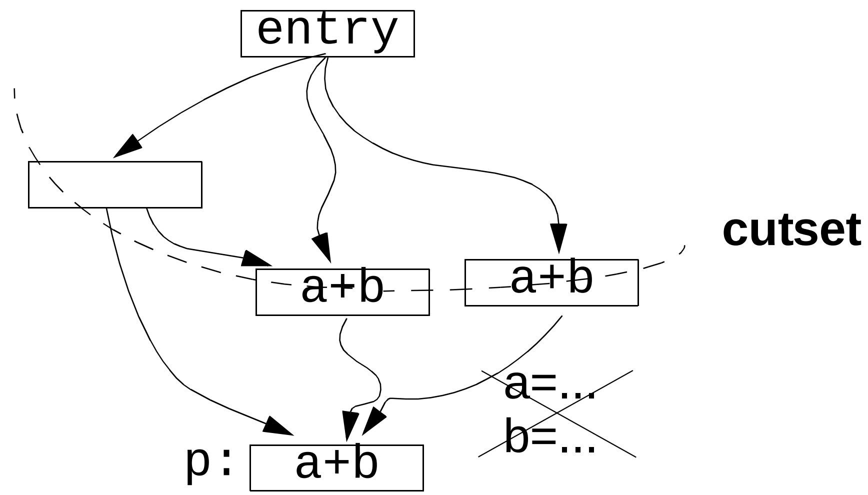
- Critical edges
  - source basic block has multiple successors
  - destination basic block has multiple predecessors
- Assume every statement is a basic block
  - Only place statements at the beginning of a basic block
  - Add a basic block for every edge
    that leads to a basic block with multiple predecessors

Full Redundancy



- **Full redundancy at p: expression a+b redundant on all paths**
  - cutset: nodes that separate entry from p
  - cutset contains calculation of a+b
  - a, b, not redefined

Partial Redundancy



- **Partial redundancy at p: redundant on some but not all paths**
    - Add operations to create a cutset containing a+b
    - Note: Moving operations up can eliminate redundancy
- **Constraint on placement: no wasted operation**
    - a+b is “anticipated” at B if its value computed at B will be used along ALL subsequent paths
    - a, b not redefined, no branches that lead to exit with out use
- **Range where a+b is anticipated --> Choice**

Pass 1: Anticipated Expressions

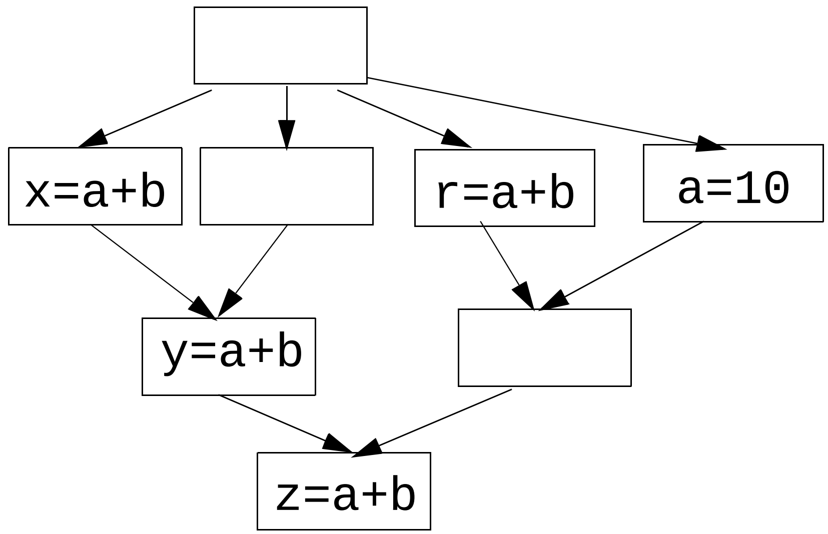
- **Backward pass: Anticipated expressions**

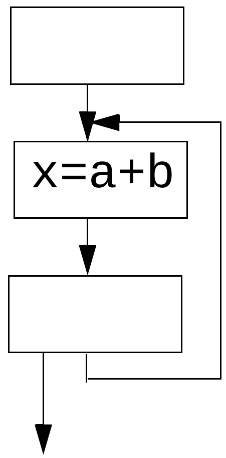
  Anticipated[b].in: Set of expressions anticipated at the entry of b

  - An expression is anticipated if its value computed at point p will be used along ALL subsequent paths

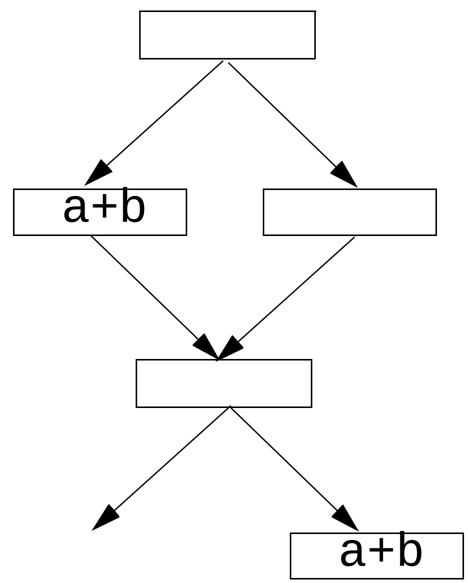
	Anticipated Expressions
Domain	Sets of expressions
Direction	backward
Transfer function	$f_b(x) = EUse_b \cup (x - EKill_b)$ EUse: used exp EKill: exp killed
$\wedge$	$\cap$
Boundary	in[exit] = $\emptyset$
Initialization	in[b] = {all expressions}

Examples (1)

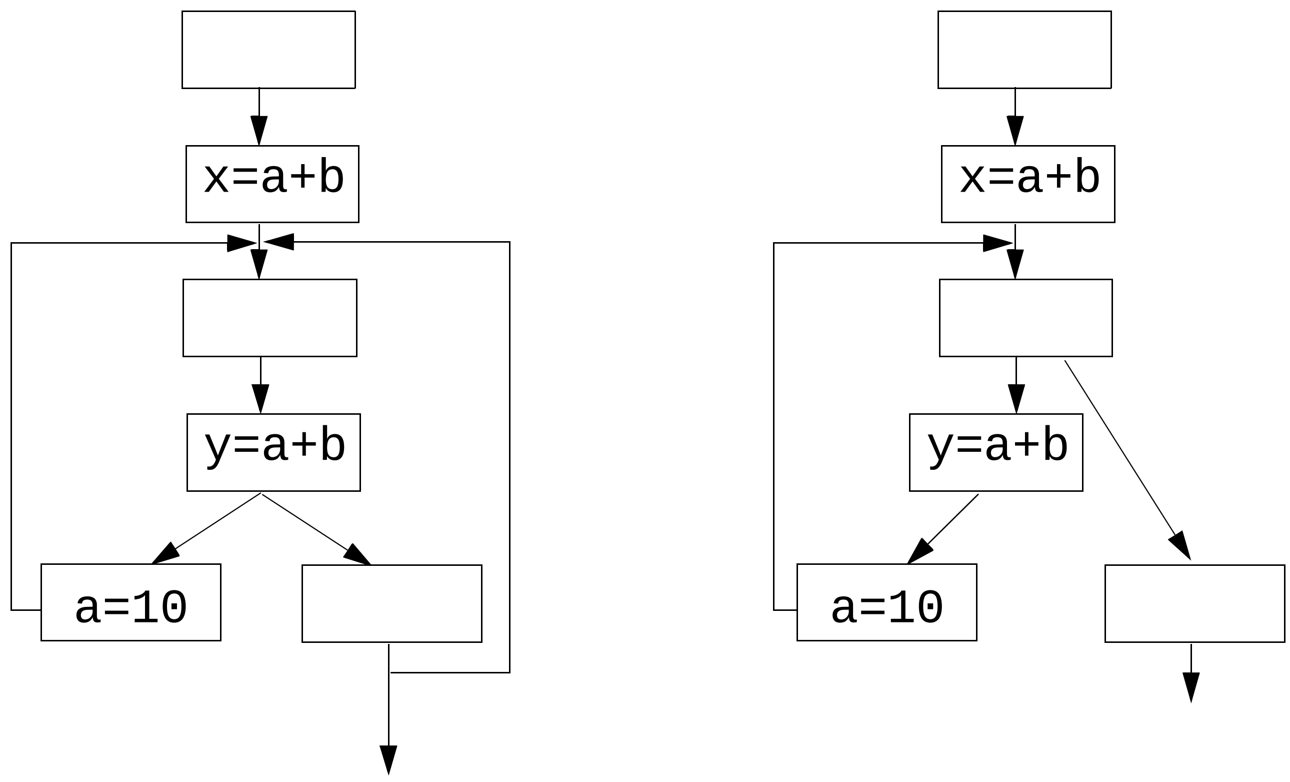




Examples (2)

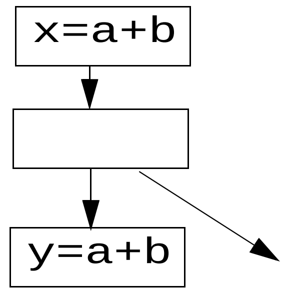


Examples (3)



Pass 2: Place As Early As Possible

- **First approximation: frontier between "not anticipated" & "anticipated"**
- **Complication: Anticipation may oscillate**



- Assume: place expression e such that it is available where it is anticipated.
- e will be **available** at p
  if e has been anticipated but not subsequently killed on all paths reaching p

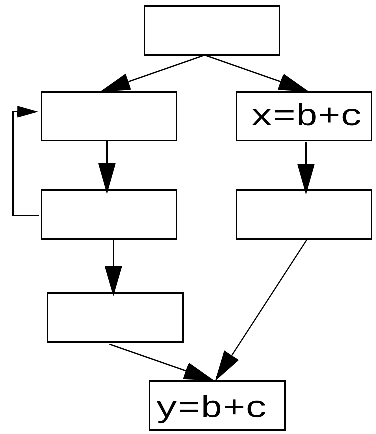
	Available Expressions
Domain	Sets of expressions
Direction	forward
Transfer function	$f_b(x) = (\text{Anticipated}[b].\text{in} \cup x) - \text{EKill}_b$
$\wedge$	$\cap$
Boundary condition	out[entry] = $\emptyset$
Initialization	out[b] ={all expressions}

Early Placement

- **earliest(b)**
  - set of expressions added to block b under early placement
- **Place expression at the earliest point anticipated and not already available**
  - earliest(b) = anticipated[b].in – available[b].in
- **Algorithm**
  - For all basic block b, if x+y $\in$ earliest[b]
    - at beginning of b:
      create a new variable t
      t = x+y,
      replace every original x+y by t

Pass 3: Lazy Code Motion

- **Delay without creating redundancy to reduce register pressure**



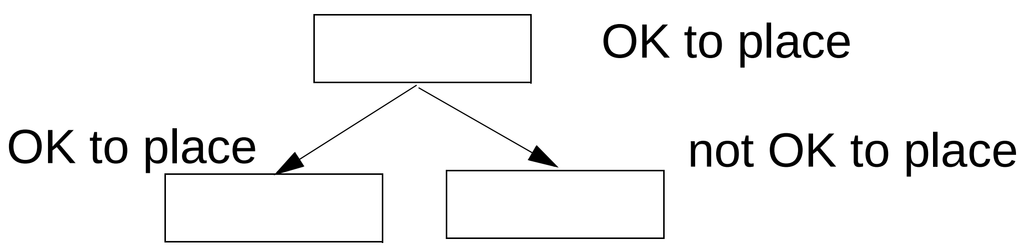
- **An expression e is postponable at a program point p if**
  - all paths leading to p
    have seen the earliest placement of e but not a subsequent use

	Postponable Expressions
Domain	Sets of expressions
Direction	forward
Transfer function	$f_b(x) = (\text{earliest}[b] \cup x) - \text{EUse}_b$
$\wedge$	$\cap$
Boundary condition	out[entry] = $\emptyset$
Initialization	out[b] = {all expressions}

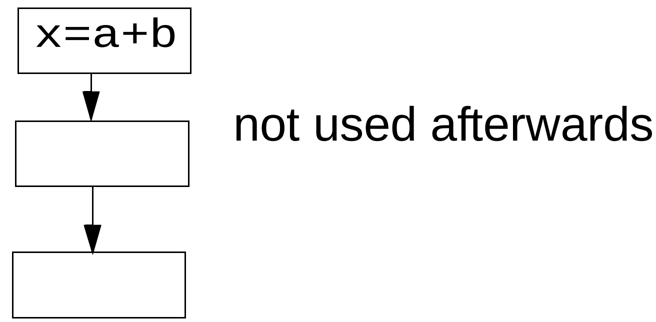
Latest: frontier at the end of “postponable” cut set

- latest[b] = (earliest[b] $\cup$ postponable.in[b]) $\cap$
  (EUse$_b$ $\cup$ $\neg$($\bigcap_{s \in succ[b]}$(earliest[s] $\cup$ postponable.in[s])))
  - OK to place expression: earliest or postponable
  - Need to place at b if either
    - used in b, or
    - not OK to place in one of its successors

- Note because of pre-processing step:
  - if one of its successors cannot accept postponement, b has only one successor
  - The following does not exist



Pass 4: Cleaning Up



- **Eliminate temporary variable assignments unused beyond current block**
- **Compute: Used.out[b]: sets of used (live) expressions at exit of b.**

	Used Expressions
Domain	Sets of expressions
Direction	backward
Transfer function	$f_b(x) = (EUse[b] \cup x)$ -latest[b]
$\wedge$	$\cup$
Boundary condition	in[exit] = $\emptyset$
Initialization	in[b] = $\emptyset$

Code Transformation

- For all basic blocks b,

  if $(x+y) \in$ (latest[b] $\cap$ used.out[b])

  at beginning of b:

  add new t = x+y

  if $(x+y) \in$ ($EUse_b \cap \neg$ (latest[b] $\cap \neg$ used.out[b]))

  replace every original x+y by t

Summary

- **Cannot execute any operations not executed originally**
    - Pass 1: Anticipation: range of code motion
- **Eliminate as many redundant calculations of an expression as possible, without duplicating code**
    - Pass 2: Availability: move it up as early as possible
- **Delay computation as much as possible to minimize register lifetimes**
    - Pass 3: Postponable: move it down unless it creates redundancy (lazy code motion)
- **Pass 4: Remove temporary assignment**

Remarks

- **Powerful algorithm**
    - Finds many forms of redundancy in one unified framework
- **Illustrates the power of data flow**
    - Multiple data flow problems